Name: Solutions

This assignment consists of seven questions, each worth five points for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help. Remember to start this assignment early, your next quiz is based on this assignment.

1. Differentiate the following functions.

(a) $y = 16(9x^3 - 4x + 5)^7$ Chain Rule

$u(x) = 16x^7$ $u'(x) = 112x^6$

$v(x) = 9x^3 - 4x + 5$ $v'(x) = 27x^2 - 4$

$y = u(v(x))$ $y' = v'(x)u'(v(x))$

Answer: $y' = (27x^2-4)112(9x^3-4x+5)^6$

(b) $y = \ln(5x^2)(3x^2 - 5)$ Product Rule

$u(x) = \ln(5x^2)$ ← chain Rule: $a(x) = \ln(x)$ $a'(x) = \frac{1}{x}$

$b(x) = 5x^2$ $b'(x) = 10x$

$V(x) = 3x^2 - 5$ $V'(x) = 6x$ $u(x) = a(b(x))$ $u'(x) = b'(x)a'(b(x))$

$= 10x \cdot \frac{1}{5x^2}$

$y = u(x)v(x)$ $y' = u'(x)v(x) + u(x)v'(x)$

Answer: $y' = 10x \cdot \frac{1}{5x^2}(3x^2-5) + \ln(5x^2)6x$

$= \frac{2}{x}(3x^2-5) + 6x\ln(5x^2)$

(c) $y = \dfrac{3x^2 + 19x - 5}{3e^x}$ Product Rule

$v(x) = (3e^x)^{-1} = \frac{1}{3}e^{-x}$ ← chan rule $a(x) = \frac{1}{3}e^x$ $a'(x) = \frac{1}{3}e^x$

$b(x) = -x$ $b'(x) = -1$

$u(x) = 3x^2 + 19x - 5$ $u'(x) = 6x + 19$ $v(x) = a(b(x))$ $v'(x) = b'(x)a'(b(x))$

$= -\frac{1}{3}e^{-x}$

$y = u(x)v(x)$ $y' = u'(x)v(x) + u(x)v'(x)$

Answer: $y' = (6x+19)\frac{1}{3}e^{-x} + (3x^2+19x-5)(-\frac{1}{3}e^{-x})$

2. (a) What is the rate of change of the function $f(x) = 3x - 7$?

Answer: $f'(x) = 3$

(b) What is the rate of change of the function $g(x) = 4e^{3x}$?

Chain rule

Answer: $g'(x) = 12e^{3x}$

(c) What is the relative rate of change of $f(x)$?

$$\frac{f'(x)}{f(x)}$$

Answer: $\frac{3}{3x-7}$

(d) What is the relative rate of change of $g(x)$?

$$\frac{g'(x)}{g(x)}$$

Answer: 3

3. (a) Some antique furniture increased very rapidly in price over the past decade. For example, the price of a particular rocking chair is well approximated by $V(t) = 75(1.35)^t$, where V is in dollars and t is in years since 2000. Find the rate, in dollars per year, at which the price is increasing at time t.

Answer: $V'(t) = 75\ln(1.35)(1.35)^t$

(b) In 2009 a particular survey claimed the population of the world P, in billions, is approximately $P = 6.8e^{0.012t}$, where t is the number of years since 2009. At what rate was the world's population increasing by on that date? Give your answer in millions of people per year.

$$P' = 6.8(0.012)e^{0.012t}$$

$$P'(0) = 6.8(0.012) = 0.0816$$

Answer: 81.6 million people per year

Cont.

4. The temperature, H, in degrees Fahrenheit ($^\circ F$) of a can of soda that is put into a refrigerator to cool is given by $H(t) = 40 + 30e^{-2t}$, where t is the number of hour since it was placed in the refrigerator.

   (a) Find the rate at which the temperature of the soda is changing (in $^\circ F$/hour).

   Answer: $H'(t) = -60e^{-2t}$

   (b) What is the sign of $\frac{dH}{dt}$? Interpret this.

   negative : The temperature is decreasing

   (c) When, for $t \geq 0$, is the magnitude of $\frac{dH}{dt}$ largest? Why do you think this is?

   $t=0$, $H'(0) = -60$

   $|H'(0)| = 60$

   : The temperature decreases fast as soon as its put in the fridge. This rate slows as the temperature of the can reaches that of the fridge

   (d) What is the lowest temperature the can of soda can reach?

   Answer: $40^\circ F$

   (e) Why is this the lowest temperature?

   Its the temp. of the fridge. You cannot decrease past this temp

   "or" The temp. is decreasing and $30e^{-2t} > 0$. So as t gets big $30e^{-2t}$ gets small, but H can never go below 40.

5. Is the function $f(x) = x^5 - 3x^4 + x^3 - x^2 + 10x + 17$ concave up, down or neither at the point $x = 3$?

$$f'(x) = 5x^4 - 12x^3 + 3x^2 - 2x + 10$$
$$f''(x) = 20x^3 - 36x^2 + 6x - 2$$
$$f''(3) = 20(3)^3 - 36(3)^2 + 6(3) - 2$$
$$= 20(27) - 36(9) + 18 - 2$$
$$= 540 - 324 + 16$$
$$= 216 + 16$$
$$= 232 > 0$$

Answer: Concave up

Cont.

6. (a) Find the *slope* of the tangent line to $f(x) = 3x^2 - 6x + 7$ at the point $x = 3$.

$f'(x) = 6x - 6$

$f'(3) = 6(3) - 6$

$= 18 - 6$

$= 12$

Answer: $m = 12$

(b) Find the *equation* of the tangent line to $f(x) = 3x^2 - 6x + 7$ at the point $x = 3$.

$f(3) = 3(3)^2 - 6(3) + 7 = 27 - 18 + 7 = 16$

$y - 16 = 12(x - 3)$

Answer: $y = 12(x-3) + 16$

7. Find the equation of the tangent line to the graph $f(x) = 1 - e^x$ at the point where $f(x)$ crosses the x-axis.

$1 - e^x = 0 \implies 1 = e^x \implies \ln(1) = x \implies 0 = x$

$Pt = (0, 0)$

$f'(x) = -e^x$

$f'(0) = -e^0 = -1$

$y - 0 = -1(x - 0)$

$m = -1$

Answer: $y = -x$

The End.